

Dynamics of Rolling Massive Scalar Field Cosmology

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We study the inflationary consequences of the rolling massive scalar field in the braneworld scenario with a warped metric. We find that in order to fit observational constraints the warp factor must be tuned to be $< 10^{-3}$. We also demonstrate the inflationary attractor behavior of the massive scalar field dynamics both in the standard FRW case as well as in braneworld scenario.

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I. INTRODUCTION

The inflationary paradigm [1], despite overwhelming support from observational data [2], is still ad hoc. There still is no conclusive model supported by a fundamental theory which provides a mechanism for its realization. It is hoped that M/string theory may provide the theoretical framework for realization of inflation and there has recently been a lot of activity towards construction of models of inflation in string theory. See [3] for a review. Originating from the work of Sen [4], the possibility of the tachyon field being a candidate for the inflaton has been extensively studied [5]. The tachyon action is of the Dirac Born Infeld form [6] which leads to an equation of state interpolating between -1 at early times and 0 at late times. This suggests the possibility that the tachyon can play the role of the inflaton in early times and dark matter at late times. However tachyonic inflation is found to be plagued by serious difficulties, namely, large density perturbations, problem with reheating and formation of caustics [7].

In a recent paper [8] Garousi, Sami and Tsujikawa put forward an inflationary model provided by a DBI type effective field theory of rolling massive scalar field on a D brane or anti- D brane, obtained from string theory [9]. Not surprisingly, as in the case of tachyon inflation the amplitude of density perturbations obtained are too large. However by considering the D brane in a warped background, the warping factor is introduced in the model as a parameter that can be tuned to get acceptable density perturbations. The authors demonstrate that the inflationary observables in the model, such as the spectral index and tensor-to-scalar ratio, can be fitted within the observational constraints, thus making it a viable model for inflation. Typically a very small value of the warp factor, of the order of 10^{-8} is required to fit the cosmological constraints. Fluxes are the sources of warping in flux compactification and it has been shown

that a very small value of β is indeed possible for suitable choices of fluxes [10].

In this paper we study the cosmological consequences of the rolling massive scalar field on a (4+1)-dimensional brane world scenario. The motivation is to see whether the enhancement in inflation due to the brane correction term in the Friedmann equation renders it possible to have a viable inflationary model without tuning the warp factor to be small. We find that a small value of the warp factor is still a necessity so as to get correct amount of density perturbations. However its order of magnitude must be 10^{-3} in contrast to the 10^{-8} in standard FRW cosmology.

We also study the phase space behavior of the rolling massive scalar field in standard FRW cosmology and in the braneworld scenario. We demonstrate that the phase trajectories rapidly converge towards the fixed point thereby exhibiting attractor behavior.

II. REVIEW OF MASSIVE ROLLING SCALAR FIELD INFLATION IN WARPED COMPACTIFICATION

Consider a BPS-D3-brane in type IIB string theory where the six extra dimensions are compactified. The effective action for the rolling massive scalar field obtained from string theory is given by

$$S = - \int d^4x V(\phi) \sqrt{-\det(\eta_{ab} + \partial_a \phi \partial_b \phi)} \quad (1)$$

where ϕ has the unit of $\sqrt{\alpha'}$, α' being the string length. The mass of the field ϕ is given by $m^2 = (n-1)/\alpha'$, n being an integer and so for $n \geq 2$ the scalar fields are massive.

The potential of the scalar field, to fourth order in ϕ , was derived [9] to be

$$V(\phi) = \tau_3 \left(1 + \frac{1}{2} m^2 \phi^2 + \frac{1}{8} m^4 \phi^4 + \dots \right) \quad (2)$$

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which in ref [8] is speculated to have the closed form

$$V(\phi) = \tau_3 e^{\frac{1}{2} m^2 \phi^2} \quad (3)$$

Consider the warped metric

$$ds_{10}^2 = \beta(y_i) \eta_{ab} dx^a dx^b + \beta^{-1}(y_i) g_{ij} dy^i dy^j \quad (4)$$

where $\beta(y_i)$ is the warping factor depending only on the coordinates of the compact directions y_i and \tilde{g}_{ij} is the metric on the compact space. Let us consider a scenario in which the brane can move in the compact space thereby reducing its tension. This corresponds physically to the situation where the warp factor β is small at some point on the compact space. The action for a massive scalar field rolling on the D_3 -brane minimally coupled to gravity is then given by the DBI form

$$S = - \int d^4x \beta^2 V(\phi) \sqrt{-\det(\eta_{ab} + \beta^{-1} \partial_a \phi \partial_b \phi)} \quad (5)$$

Redefining the scalar field as $\phi \rightarrow \sqrt{\beta} \phi$ we get

$$S = - \int d^4x V(\phi) \sqrt{-\det(\eta_{ab} + \partial_a \phi \partial_b \phi)} \quad (6)$$

with the redefined potential

$$V(\phi) = V_0 e^{\frac{1}{2} m^2 \beta \phi^2}, \quad V_0 = \beta^2 \tau_3 \quad (7)$$

In a spatially flat FRW background with a scale factor a , the energy density ρ and the pressure p of the field are given by

$$\rho = \frac{V}{\sqrt{1 - \dot{\phi}^2}} \quad (8)$$

$$p = -V \sqrt{1 - \dot{\phi}^2} \quad (9)$$

The inflaton equation of motion which follows from (7) is

$$\frac{\ddot{\phi}}{1 - \dot{\phi}^2} + 3H\dot{\phi} + \frac{V'}{V} = 0 \quad (10)$$

where the prime denotes differentiation with respect to ϕ . The Friedmann equation is

$$H^2 = \frac{\kappa^2}{3} \frac{V}{\sqrt{1 - \dot{\phi}^2}} \quad (11)$$

The slowroll parameters are

$$\epsilon = \frac{M_P^2}{2} \frac{V'^2}{V^3}, \quad \eta = -2\epsilon + M_P^2 \frac{V''}{V^2} \quad (12)$$

For the potential given by (7) they take the form

$$\epsilon = \frac{m^4 M_P^2}{2\tau_3} \phi^2 e^{-\frac{1}{2} m^2 \beta \phi^2} \quad (13)$$

$$\eta = \frac{m^2 M_P^2}{\beta \tau_3} e^{-\frac{1}{2} m^2 \beta \phi^2} \quad (14)$$

The slow roll condition $\epsilon < 1$ and $|\eta| < 1$ is trivially satisfied by choosing sufficiently large initial ϕ . The number of e-folding during inflation is given by

$$N = \int_{t_i}^{t_{\text{end}}} H dt. \quad (15)$$

By choosing initial ϕ large enough it is always possible to get sufficient number of efoldings.

The amplitude of the density perturbation is given by [11]

$$\mathcal{P}_s \sim \frac{H^4}{(2\pi\dot{\phi})^2 V} \quad (16)$$

which in the slow roll approximation becomes

$$\mathcal{P}_s \simeq \frac{1}{12\pi^2 M_P^2} \left(\frac{V^2}{V'} \right)^2 \quad (17)$$

and for the potential (7) it becomes

$$\mathcal{P}_s \simeq \frac{\beta^2}{48\pi^2 \epsilon^2} \frac{m^6 \phi^2}{M_P^2} \quad (18)$$

Observational constraints demand that $\mathcal{P}_s \simeq 10^{-9}$, from which we can get an estimate on the range of β . Taking $m^2 \beta \phi^2 \sim 1$ and $\epsilon \sim 1$ and putting $m/M_P \sim 1$ we get $\beta < 10^{-8}$. Of course for getting the correct estimate of β we must put the values of ϕ and ϵ at fifty efolds or so, in which case β will have to be much less than the above estimate. Here we are only highlighting the fact that β has to be tuned to be much less than one to fit the observational constraints. Thus we see that in the absence of warping, that is for $\beta = 1$, it is impossible to get a viable model of inflation from the rolling massive scalar field.

III. ROLLING SCALAR FIELD ON THE BRANE

In the braneworld scenario the prospects of inflation become highly favourable due to high energy corrections to the Friedmann equation.

$$H^2 = \frac{1}{3M_P^2} \rho \left(1 + \frac{\rho V}{\lambda_b} \right) \quad (19)$$

where λ_b is the brane tension. In the high energy limit we have $\rho/\lambda_b \gg 1$ and inflation occurs as long as $\dot{\phi}^2 < 1/3$. In this section we analyze inflation for the rolling massive scalar field in the braneworld scenario to see whether we still require to tune the warp factor to get the right amount of density perturbations.

The slow roll parameters take the form

$$\epsilon = 12M_P^2\lambda_b\left(\frac{V'}{V^2}\right)^2 \quad (20)$$

$$\eta = -6M_P^2\lambda_b\frac{V''}{V^3} \quad (21)$$

The slow roll condition can be satisfied by choosing large initial ϕ . The number of e-folds is

$$\begin{aligned} N &= -\frac{1}{2M_P^2\lambda_b}\int_{\phi_i}^{\phi_f}\frac{V^3}{V'}d\phi \\ &= -\frac{1}{4}\frac{V_0^2}{M_P^2\lambda_b m^2\beta}\int_{x_i}^{x_f}\frac{e^x}{x}dx \end{aligned}$$

where $x = m^2\beta\phi^2$. Again sufficient number of e-foldings can always be obtained by choosing large initial ϕ . The amplitude of density perturbations is

$$\begin{aligned} \mathcal{P}_s &= \frac{9}{4\pi^2(6M_P^2\lambda_b)^3}\frac{V^7}{V'^2} \\ &= \frac{3}{2\pi^2}\frac{\beta^2 V_0}{M_P^2\lambda_b}\frac{m^4\phi^2}{\epsilon^2}e^{\frac{1}{2}m^2\beta\phi^2} \end{aligned}$$

To estimate β we put $m^2\beta\phi^2 \sim 1$ and $\epsilon \sim 1$ and $\tau_3/(m^2M_P^2) \sim 1$ which gives

$$\mathcal{P}_s \simeq \frac{3}{2\pi^2}\frac{\beta^3}{\lambda_b}m^4 \quad (22)$$

Putting $\mathcal{P}_s \simeq 10^{-9}$ we get

$$\beta^3 \leq \frac{\lambda_b}{m^4}10^{-9} \quad (23)$$

Since the brane tension λ_b must be $\ll M_P^4$ we have

$$\beta^3 \ll \left(\frac{M_P}{m}\right)^4 10^{-9} \quad (24)$$

which on taking $m/M_P \sim 1$ gives $\beta \ll 10^{-3}$. Thus in the braneworld scenario we still crucially require small value for β despite the enhanced value of the Hubble parameter H coming from the brane correction.

IV. INFLATIONARY ATTRACTOR FOR DBI MASSIVE SCALAR FIELD

In this section we demonstrate the attractor behavior of rolling scalar field inflation cosmology in standard FRW and brane world scenarios. Introducing the dimensionless variables

$$x \equiv \dot{\phi}, \quad y \equiv \frac{V}{V_0}, \quad \tau \equiv mt \quad (25)$$

we can write Eq.(10), subject to the constraint imposed by the Friedmann equation, as the following two coupled equations

$$\begin{aligned} x' &= -\sqrt{\frac{3V_0}{m^2M_P^2}}\sqrt{y}x(1-x^2)^{3/4} - \sqrt{2\beta\ln y}(1-x^2) \\ y' &= \sqrt{2\beta}xy\sqrt{\ln y} \end{aligned} \quad (26)$$

where prime denotes differentiation with respect to τ in this section. We get the fixed points $(0, 1)$ and $(\pm 1, 1)$. The latter two points are not physically interesting because they can be removed by another choice of coordinates, say $x \equiv \dot{\phi}$ and $y \equiv \phi$. In order to see the stability of the critical point $(0, 1)$ we perturb about the fixed point

$$\begin{aligned} x &= \epsilon \\ y &= 1 + \delta \end{aligned}$$

where ϵ and δ are infinitesimally small. Putting them in eqs. (26) and keeping only lowest order terms we get

$$\begin{aligned} \epsilon' &= -\sqrt{\frac{3V_0}{m^2M_P^2}}\epsilon + \sqrt{2\beta\delta} \\ \delta' &= \sqrt{2\beta}\delta\epsilon \end{aligned}$$

Defining $\gamma^2/2 \equiv \delta$ we can linearize the above equations to get

$$\begin{aligned} \epsilon' &= -\sqrt{\frac{3V_0}{m^2M_P^2}}\epsilon + \sqrt{\beta\gamma} \\ \gamma' &= \sqrt{\beta}\epsilon \end{aligned}$$

which can be solved in matrix form to get

$$\begin{pmatrix} \epsilon \\ \gamma \end{pmatrix} = \begin{pmatrix} \epsilon_0 \\ \gamma_0 \end{pmatrix} e^{R\tau} \quad (27)$$

where R is given by

$$\begin{pmatrix} -\sqrt{\frac{3V_0}{m^2M_P^2}} & -\sqrt{\beta} \\ 1 & 0 \end{pmatrix} \quad (28)$$

having the eigenvalues

$$\lambda_{\pm} = -\frac{1}{2}\sqrt{\frac{3V_0}{m^2M_P^2}} \pm \sqrt{\frac{3V_0}{m^2M_P^2} - 4\sqrt{\beta}} \quad (29)$$

It is clear that the real part of λ_{\pm} is always negative and so the fixed point $(0, 1)$ is stable. As can be seen from the numerical solution of Eq.(26) shown in Fig.(1) trajectories originating at any initial point on the phase space end up at the stable fixed point $(0, 1)$.

We next study the phase space behavior in the braneworld scenario. On the brane Eqs.(26) gets modified to

$$\begin{aligned} x' &= -\sqrt{\frac{3V_0}{m^2M_P^2}}\sqrt{\frac{y}{\sqrt{1-x^2}}}\left(1 + \frac{V_0}{\lambda_b}\frac{y}{\sqrt{1-x^2}}\right)x(1-x^2) \\ &\quad - \sqrt{2\beta\ln y}(1-x^2) \end{aligned} \quad (30)$$

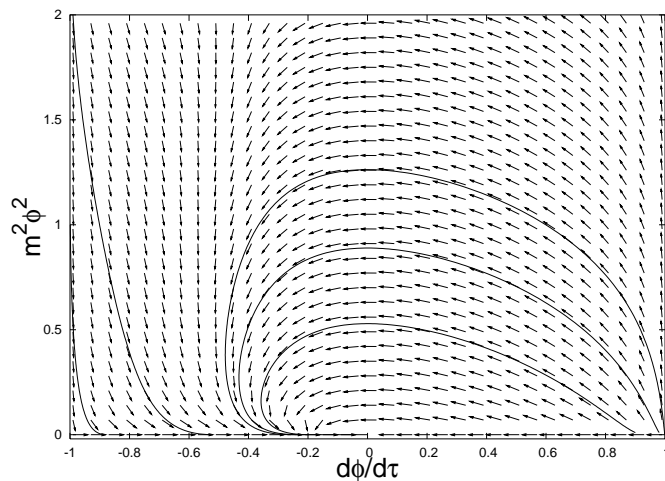


FIG. 1: Phase plot of rolling massive scalar field in standard FRW cosmology. Trajectories approach the stable fixed point (0,1).

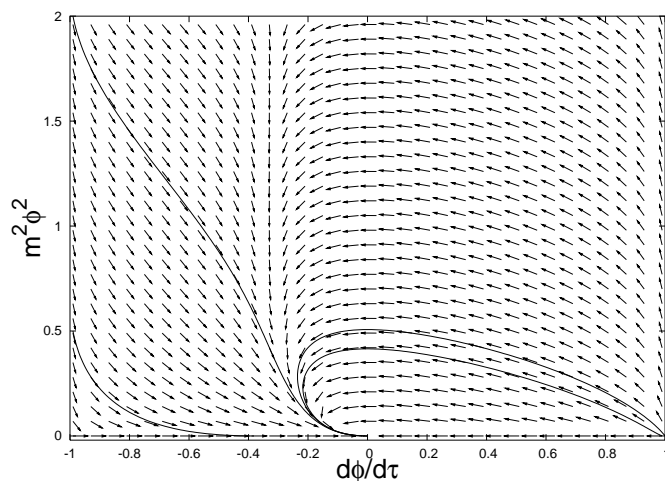


FIG. 2: Plot of phase trajectories for rolling massive scalar field in braneworld. As in the previous figure, trajectories approach the stable fixed point (0,1) but at a faster rate.

The fixed points are the same as the previous case. The stability analysis can be carried out as before. The phase trajectories are shown in fig. (2). All trajectories are seen to move toward the stable fixed point (0,1).

From a comparison of the trajectories in Figs. (1) and (2) we see that they approach the stable fixed point (0,1) faster in the braneworld case. This is a consequence of the enhancement of the friction term in the equation of

motion of ϕ due to the high energy correction in the Friedmann equation.

V. CONCLUSION

In this paper we have shown that for the rolling massive scalar field to be a good candidate for the inflaton in a brane world scenario, tuning of the warping factor to be small is still crucially necessary, despite the enhancement of inflation coming from the brane correction term. The order of magnitude of the warping factor is however much improved. The difficulty with tachyonic inflation, namely that of large density perturbations, in conventional compactification is also faced by the rolling massive scalar field inflation even though the potentials in the two models are vastly different. The reason is, the two parameters in the model, namely, the string coupling g_s and the compactification volume v are constrained by string theory to be $g_s < 1$ and $v \gg 1$, which is incompatible with the requirement for small density perturbations. It must be pointed out that while warped compactification solves the problem for the rolling massive scalar field it does not do so for the tachyon. The slow roll condition for the tachyon, in conventional compactification, is given in terms of g_s and v as roughly $g_s/v \gg 127$. In the warped compactification this is modified to $g_s\beta/v \gg 127$ which cannot be satisfied for small β . However, in a warped compactification and varying g_s the tachyon may still be a good candidate for the inflaton as shown recently in [12].

We have further studied the dynamics of the rolling scalar field. In particular we have demonstrated the attractor behavior of inflation driven by this field in standard FRW cosmology and brane world cosmology. The trajectories show attractor behavior and rapidly converge towards the fixed point. Thus the evolution of the field, and consequently the onset of inflation, is independent of the initial conditions one may choose on the phase space, implying that the predictions of the physical observables, such as the amplitude of density perturbations are indeed independent of the initial conditions.

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